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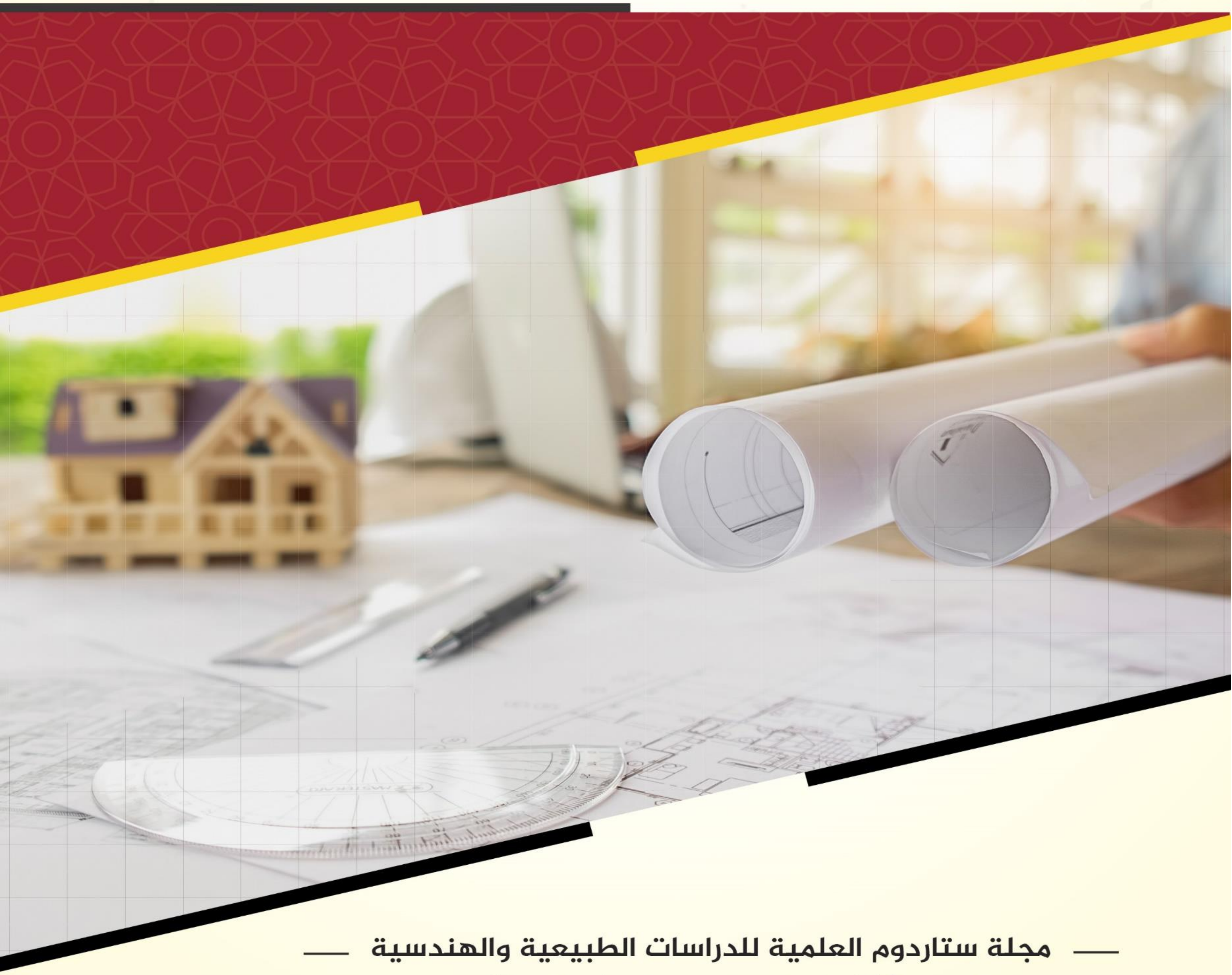

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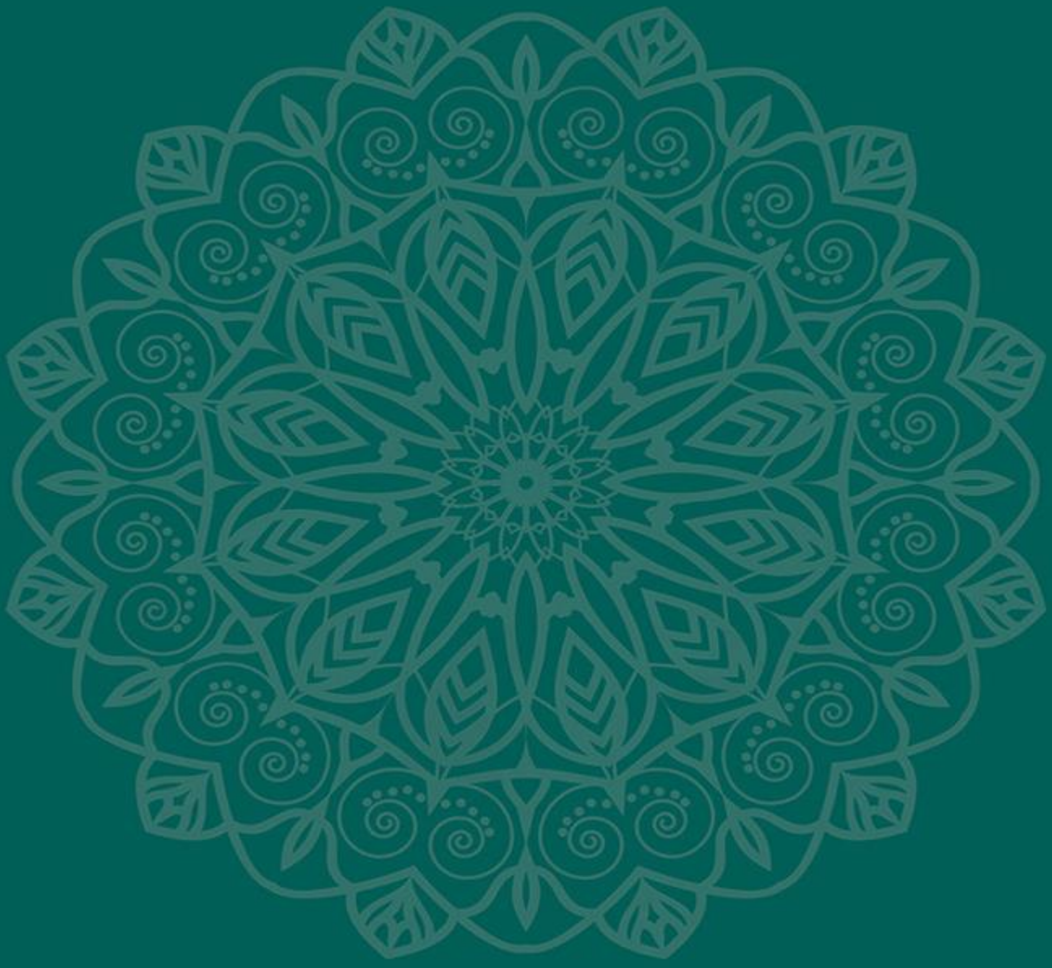

Peer Reviewing Committee of Stardom Scientific Journal of Natural and Engineering Sciences

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عناوين الأبحاث

Stardom Journal of Natural and Engineering Sciences (SJNES)

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[2] W. Strunk Jr., E.B. White, The Elements of Style, fourth ed., Longman, New York, 2000.

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[3] G.R. Mettam, L.B. Adams, How to prepare an electronic version of your article, in: B.S. Jones, R.Z. Smith (Eds.), Introduction to the Electronic Age, E-Publishing Inc., New York, 2009, pp. 281–304.

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Bilateral Generating Functions for the Two-Parameter Three-variable Srivastava polynomials

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Abstract:

In this paper, we prove ageneral theorems on generating functions involving the two-parameter three-variable Srivastava polynomials, Hermite polynomials and Legendre Polynomials of pseudo two variables. Some applications of these theorems lead us to derive several bilateral generating functions involving some well-known classical polynomials of one variable which are contained by the two-parameter three -variable Srivastava polynomials.

MSC 2010 :33C45**,** 33C05**,** 33C65**.**

Keywords: Generating functions, Srivastava polynomials, Hermite polynomials, Legendre Polynomials.

1. Introduction

In 1972, Srivastava [8] introduced the following family of polynomials:

$$S_n^N(x)=\sum_{k=0}^{[\frac{n}{N}]}\frac{(-n)_{Nk}}{k!}A_{n,k}\,x^k \quad (n\in\mathbb{N}_0=\mathbb{N}\cup\{0\};N\in\mathbb{N})\,, \qquad (1.1)$$

where $\mathbb{N}$ is the set of positive integers, $\{A_{n,k}\}_{n,k=0}^{\infty}$ is a bounded double sequence of real or complex numbers, $[a]$ denotes the greatest integer of $a\in\mathbb{R}$ and $(\lambda)_n$ denotes the Pochhammer symbol defined by [9]

$$(\lambda)_n=\frac{\Gamma(\lambda+n)}{\Gamma(\lambda)}, \quad \lambda\neq 0,-1,-2,\cdots \qquad (1.2)$$

where $\Gamma(.)$is Gamma function.

In 2001, Gonzalez *et al.* [1] extended the Srivastava polynomials $S_n^N(x)$ as follows:

$$S_{n,m}^N(x)=\sum_{k=0}^{[\frac{n}{N}]}\frac{(-n)_{Nk}}{k!}A_{n+m,k}\,x^k \quad (n,m\in\mathbb{N}_0;\ N\in\mathbb{N})\,. \qquad (1.3)$$

In 2013, Kaanoglu and Ozarslan [4] introduced the following family of one-variable, two-parameter and three-variables Srivastava polynomials as follow:

$$S_n^{p,q}(x)=\sum_{k=0}^{n}\frac{(-n)_k}{k!}A_{p+q+n,q+k}\,x^k \quad (p,q,n,k\in\mathbb{N}_0)\,, \qquad (1.4)$$

In [4], the following family of bivariate polynomials was introduced:

$$S_n^{p,q}(x,y) = \sum_{k=0}^{n} A_{p+q+n,q+k} \frac{x^k}{k!} \frac{y^{n-k}}{(n-k)!} \quad (p,q,n,k \in \mathbb{N}_0, k \le n), \tag{1.5}$$

where $\{A_{n,k}\}$ is a bounded double sequence of real or complex numbers.

In [10], Srivastava et al. introduced the three-variable polynomials

$$S_n^{p,q,M}(x,y,z) = \sum_{k=0}^{n} \sum_{l=0}^{[k/M]} A_{p+q+n,q+k,l} \frac{x^l}{l!} \frac{y^{k-Ml}}{(k-Ml)!} \frac{z^{n-k}}{(n-k)!}, \tag{1.6}$$

$$\left(p,q,n,k,l \in \mathbb{N}_0, M \in N,\ Ml \le k \le n\right)$$

where $\{A_{n,k,l}\}$ is a triple sequence of complex numbers. Suitable choices of $\{A_{n,k,l}\}$ in equation (1.6) give a three-variable version of well-known polynomials (see also [2]. Re-cently, in [3], the multivariable extension of the Srivastava polynomials in r-variable was introduced

$$S_n^{m,N_1,N_2,\ldots,N_{r-1}}(x_1,x_2,\ldots,x_r) := \sum_{k_{r-1}=0}^{\left[\frac{n}{N_{r-1}}\right]} \sum_{k_{r-2}=0}^{\left[\frac{k_{r-1}}{N_{r-2}}\right]} \cdots \sum_{k_2=0}^{\left[\frac{k_3}{N_2}\right]} \sum_{k_1=0}^{\left[\frac{k_2}{N_1}\right]} A_{m+n,k_{r-2},k_1,k_2,\ldots,k_{r-1}}$$

$$\frac{x_1^{k_1}}{k_1!} \frac{x_2^{k_2-N_1k_1}}{(k_2-N_1k_2)!} \cdots \frac{x_r^{n-N_{r-1}k_{r-1}}}{(n-N_{r-1}k_{r-1})!} \tag{1.7}$$

$$(m,n \in \mathbb{N}_0;\ N_1,N_2,\ldots,k_{r-1} \in \mathbb{N})$$

where $\{A_{m,k_{r-2},k_1,k_2,\ldots,k_{r-1}}\}$ is a sequence of complex numbers.

The Hermite polynomials of two variables are defined by [6]

$$H_n(x,y) = \sum_{r=0}^{[n/2]} \frac{(-1)^r H_{n-2r}(x)\, x^{2r}\, y^{n-2r}}{r!\ (n-2r)!} \tag{1.8}$$

where $H_n(x)$ is the well-known Hermite polynomials [7].

Also, we note that the Hermite polynomials of two variables are satisfy the following

$$\sum_{n=0}^{\infty} \frac{(c)_n \, H_n(x,y) \, t^n}{n!}$$

$$= (1-2xyt)^{-c} \, F^{2:0;0}_{0:0;0}\left[\begin{matrix} \frac{c}{2}, \frac{c}{2}+\frac{1}{2} : -; -; \\ - \quad : -; -; \end{matrix} \; \frac{-4x^2t^2}{(1-2xyt)^2}, \frac{-4y^2t^2}{(1-2xyt)^2}\right], \qquad (1.9)$$

where $F^{A:B;D}_{E:G;H}[x,y]$ is the Kampè de Fèriet function of two variables [9].

The Legendre Polynomials $P_n(x,y)$ of pseudo two variables are defined by [5]

$$P_n(x,y) = \sum_{r=0}^{\left[\frac{n}{2}\right]} \frac{n!(x^2-y-1)^r \, x^{n-2r}}{2^{2r}(r!)^2(n-2r)!} \qquad (1.10)$$

and satisfy the following generating relation [5] :

$$\sum_{n=o}^{\infty} \frac{(c)_n P_n(x,y) t^n}{n!} = (1-xt)^{-c} \; {}_2F_1\left[\begin{matrix} \frac{c}{2}, \frac{c}{2}+\frac{1}{2} \; ; \\ 1 \quad ; \end{matrix} \frac{t^2(x^2-y-1)}{(1-xt)^2}\right], \qquad (1.11)$$

where ${}_2F_1$ is the Gaussian hypergeometric function defined by [9]

$${}_2F_1\left[\begin{matrix} a, \, b; \\ c \;\; ; \end{matrix} x\right] = \sum_{n=0}^{\infty} \frac{(a)_n \, (b)_n}{(c)_n} \frac{x^n}{n!}, \quad c \neq 0, -1, -2, \cdots . \qquad (1.12)$$

Suppose also that two-parameter two-variable polynomials $P^M_{p,q}(x\,,y)$ are defined by

$$P^M_{m_1,m_2}(x,y) = \sum_{k=0}^{[m_2/M]} A_{m_1+m_2,m_2,k} \, \frac{x^{m_2-Mk}}{(m_2-Mk)\,!} \, \frac{y^k}{k!} \, , (Ml \leq \, m_2). \qquad (1.13)$$

2. Main Results

Theorem 2.1. The following family of bilateral generating functions holds true:

$$\sum_{p,q,n=0}^{\infty} H_{p+q+n}\,(u,v)\, S_n^{p,q,M}(x,y,z)\ \frac{{w_1}^p}{p!}\ \frac{{w_2}^q}{q!}\ t^n$$

$$= \sum_{p,q,Ml=0}^{\infty} H_{p+q+Ml}\,(u,v)\, A_{p+q+Ml,q+Ml,l}\ \frac{(xt^M)^l}{l!}\ \frac{(w_1+zt)^p}{p!}\ \frac{(w_2+yt)^q}{q!}. \qquad (2.1)$$

Proof: Denoting the left hand side of (2.1) by Δ, expressing $S_n^{p,q,M}(x,y,z)$ as in (1.6), we obtain

$$\Delta = \sum_{p,q,n=0}^{\infty} H_{p+q+n}\,(u,v) \sum_{k=0}^{n} \sum_{l=0}^{[k/M]} A_{p+q+n,q+k,l}\ \frac{x^l}{l!}\ \frac{y^{k-Ml}}{(k-Ml)!}\ \frac{z^{n-k}}{(n-k)!}\ \frac{{w_1}^p}{p!}\ \frac{{w_2}^q}{q!}\ t^n$$

Let $n \to n+k$

$$\Delta = \sum_{p,q,n,k=0}^{\infty} H_{p+q+n+k}\,(u,v) \sum_{l=0}^{[k/M]} A_{p+q+n+k,q+k,l}\ \frac{x^l}{l!}\ \frac{y^{k-Ml}}{(k-Ml)!}\ \frac{z^n}{n!}\ \frac{{w_1}^p}{p!}\ \frac{{w_2}^q}{q!}\ t^{n+k}$$

Let $k \to k+Ml$

$$\Delta = \sum_{p,q,n,k,Ml=0}^{\infty} H_{p+q+n+k+Ml}\,(u,v)\, A_{p+q+n+k+Ml,q+k+Ml,l}\ \frac{(xt^M)^l}{l!}\ \frac{(yt)^k}{k!}\ \frac{(zt)^n}{n!}\ \frac{{w_1}^p}{p!}\ \frac{{w_2}^q}{q!}$$

Let $p \to p-n$

$$\Delta = \sum_{p,q,k,Ml=0}^{\infty} H_{p+q+k+Ml}\,(u,v)\, A_{p+q+k+Ml,q+k+Ml,l}\ \frac{(xt^M)^l}{l!}\ \frac{(yt)^k}{k!}\frac{{w_2}^q}{q!}\left(\sum_{n=0}^{p} \frac{{w_1}^{p-n}}{(p-n)!}\ \frac{(zt)^n}{n!}\right)$$

$$\Delta = \sum_{p,q,k,Ml=0}^{\infty} H_{p+q+k+Ml}\,(u,v)\, A_{p+q+k+Ml,q+k+Ml,l}\frac{(w_1+zt)^p}{p!}\ \frac{(xt^M)^l}{l!}\ \frac{(yt)^k}{k!}\frac{{w_2}^q}{q!}$$

Let $q \to q-k$

$$\Delta = \sum_{p,q,Ml=0}^{\infty} H_{p+q+Ml}(u,v)\, A_{p+q+Ml,q+Ml,l} \frac{(w_1+zt)^p}{p!} \frac{(xt^M)^l}{l!} \left(\sum_{k=0}^{q} \frac{(yt)^k}{k!} \frac{{w_2}^{q-k}}{(q-k)!} \right)$$

$$\Delta = \sum_{p,q,Ml=0}^{\infty} H_{p+q+Ml}(u,v)\, A_{p+q+Ml,q+Ml,l} \frac{(xt^M)^l}{l!} \frac{(w_1+zt)^p}{p!} \frac{(w_2+yt)^q}{q!}.$$

This completes the proof of Theorem **2.1.**

In a similar manner, we also get the following result immediately.

Theorem 2.2. The following family of bilateral generating functions holds true:

$$\sum_{p,q,n=0}^{\infty} P_{p+q+n}(u,v)\, S_n^{p,q,M}(x,y,z) \frac{{w_1}^p}{p!} \frac{{w_2}^q}{q!} t^n$$

$$= \sum_{p,q,Ml=0}^{\infty} P_{p+q+Ml}(u,v)\, A_{p+q+Ml,q+Ml,l} \frac{(xt^M)^l}{l!} \frac{(w_1+zt)^p}{p!} \frac{(w_2+yt)^q}{q!}. \qquad (2.2)$$

Using (1.13) in the rite hand side of (2.1) and (2.2), we get:

$$\sum_{p,q,n=0}^{\infty} H_{p+q+n}(u,v)\, S_n^{p,q,M}(x,y,z) \frac{{w_1}^p}{p!} \frac{{w_2}^q}{q!} t^n$$

$$= \sum_{p,q,Ml=0}^{\infty} H_{p+q}(u,v) \frac{(w_1+zt)^p}{p!} P_{p,q}^{M}(w_2+yt\,, xt^M), \qquad (2.3)$$

$$\sum_{p,q,n=0}^{\infty} P_{p+q+n}(u,v)\, S_n^{p,q,M}(x,y,z) \frac{{w_1}^p}{p!} \frac{{w_2}^q}{q!} t^n$$

$$= \sum_{p,q,Ml=0}^{\infty} P_{p+q}(u,v) \frac{(w_1+zt)^p}{p!} P_{p,q}^{M}(w_2+yt\,, xt^M). \qquad (2.4)$$

Remark 2.1. If we set $M = 1$ and $A_{m,n,k} = (\alpha)_k\,(\beta)_{n-k}(\gamma)_{m-n}$ $(m, n \in N_0)$ in definition (1.13), we have

$$P^1_{m_1,m_2}(x,y) = (\gamma)_{m_1} g^{(\beta,\alpha)}_{m_2}(x,y). \tag{2.5}$$

Furthermore, choosing $M = 2$ and $A_{m,n,k} = (\alpha)_{m-n}(\gamma)_{n-2k}(\beta)_k$ $(m, n \in N_0)$ in defined (1.13), then

$$P^2_{m_1,m_2}(x,y) = (\alpha)_{m_1} h^{(\gamma,\beta)}_{m_2}(x,y), \tag{2.6}$$

where $g^{(\beta,\alpha)}_{m_2}(x,y)$ are the Lagrange polynomials given by

$$g^{(\beta,\alpha)}_{m_2}(x,y) = \sum_{k=0}^{[m_2]} (\alpha)_{m_2-l}\,(\beta)_l\, \frac{x^{m_2-l}}{(m_2-l)\,!}\, \frac{y^l}{l!}, \tag{2.7}$$

where $h^{(\gamma,\beta)}_{m_2}(x,y)$ denotes the Lagrange-Hermite polynomials given explicitly

$$h^{(\gamma,\beta)}_{m_2}(x,y) = \sum_{l=0}^{[m_2/M]} (\gamma)_{m_2-2l}\,(\beta)_l\, \frac{x^{m_2-2l}}{(m_2-2l)\,!}\, \frac{y^l}{l!}. \tag{2.8}$$

Remark 2.2. Choosing $M = 1$ in (1.6) and $A_{m,n,k} = (\alpha)_k\,(\beta)_{n-k}\,(\gamma)_{m-n}$, we get the following result:

$$S^{p,q,1}_n(x,y,z) = (\gamma)_p(\beta)_q\, g^{(\alpha,\beta+q,\gamma+p)}_n(x,y,z) \tag{2.9}$$

Remark 2.3. Choosing $M = 2$ in (1.6) and $A_{m,n,k} = (\alpha)_{m-n}(\gamma)_{n-2k}(\beta)_k$, we get the following result:

$$S^{p,q,2}_n(x,y,z) = (\alpha)_p(\gamma)_q\, u^{(\alpha+p,\beta,\gamma+q)}_n(x,y,z) \tag{2.10}$$

Now, using (2.5), (2,9) in (2.3), (2,4) and using (2.6), (2.10) in (2,3), (2.4), we have

$$\sum_{p,q,n=0}^{\infty} H_{p+q+n}(u,v)\,(\gamma)_p(\beta)_q\, g_n^{(\alpha,\beta+q,\gamma+p)}(x,y,z)\,\frac{{w_1}^p}{p!}\,\frac{{w_2}^q}{q!}\,t^n$$

$$=\sum_{p,q=0}^{\infty} H_{p+q}(u,v)\,\frac{(w_1+zt)^p}{p!}\,P_{p,q}^{1}(w_2+yt\,,xt^1), \qquad (2.11)$$

$$\sum_{p,q,n=0}^{\infty} P_{p+q+n}(u,v)\,(\gamma)_p(\beta)_q\, g_n^{(\alpha,\beta+q,\gamma+p)}(x,y,z)\,\frac{{w_1}^p}{p!}\,\frac{{w_2}^q}{q!}\,t^n$$

$$=\sum_{p,q=0}^{\infty} P_{p+q}(u,v)\,\frac{(w_1+zt)^p}{p!}\,P_{p,q}^{1}(w_2+yt\,,xt^1), \qquad (2.12)$$

and

$$\sum_{p,q,n=0}^{\infty} H_{p+q+n}(u,v)\,(\alpha)_p(\gamma)_q\, u_n^{(\alpha+p,\beta,\gamma+q)}(x,y,z)\,\frac{{w_1}^p}{p!}\,\frac{{w_2}^q}{q!}\,t^n$$

$$=\sum_{p,q=0}^{\infty} H_{p+q}(u,v)\,\frac{(w_1+zt)^p}{p!}\,P_{p,q}^{2}(w_2+yt\,,xt^2), \qquad (2.13)$$

$$\sum_{p,q,n=0}^{\infty} P_{p+q+n}(u,v)\,(\alpha)_p(\gamma)_q\, u_n^{(\alpha+p,\beta,\gamma+q)}(x,y,z)\,\frac{{w_1}^p}{p!}\,\frac{{w_2}^q}{q!}\,t^n$$

$$=\sum_{p,q=0}^{\infty} P_{p+q}(u,v)\,\frac{(w_1+zt)^p}{p!}\,P_{p,q}^{2}(w_2+yt\,,xt^2). \qquad (2.14)$$

Using (2.5) in (2.11), (2,12) and using (2.6) in (2,13), (2.14), we have

$$\sum_{p,q,n=0}^{\infty} H_{p+q+n}(u,v)\,(\gamma)_p(\beta)_q\, g_n^{(\alpha,\beta+q,\gamma+p)}(x,y,z)\,\frac{{w_1}^p}{p!}\,\frac{{w_2}^q}{q!}\,t^n$$

$$= \sum_{p,q,l=0}^{\infty} H_{p+q}(u,v)\,\frac{(w_1+zt)^p}{p!}\,(\gamma)_p g_q^{(\beta,\alpha)}(w_2+yt\,,xt), \qquad (2.15)$$

$$\sum_{p,q,n=0}^{\infty} P_{p+q+n}(u,v)\,(\gamma)_p(\beta)_q\, g_n^{(\alpha,\beta+q,\gamma+p)}(x,y,z)\,\frac{{w_1}^p}{p!}\,\frac{{w_2}^q}{q!}\,t^n$$

$$= \sum_{p,q=0}^{\infty} P_{p+q}(u,v)\,\frac{(w_1+zt)^p}{p!}(\gamma)_p g_q^{(\beta,\alpha)}(w_2+yt,xt), \qquad (2.16)$$

and

$$\sum_{p,q,n=0}^{\infty} H_{p+q+n}(u,v)\,(\alpha)_p(\gamma)_q\, u_n^{(\alpha+p,\beta,\gamma+q)}(x,y,z)\,\frac{{w_1}^p}{p!}\,\frac{{w_2}^q}{q!}\,t^n$$

$$= \sum_{p,q=0}^{\infty} H_{p+q}(u,v)\,\frac{(w_1+zt)^p}{p!}\,(\alpha)_p h_q^{(\gamma,\beta)}(w_2+yt,xt^2), \qquad (2.17)$$

$$\sum_{p,q,n=0}^{\infty} P_{p+q+n}(u,v)\,(\alpha)_p(\gamma)_q\, u_n^{(\alpha+p,\beta,\gamma+q)}(x,y,z)\,\frac{{w_1}^p}{p!}\,\frac{{w_2}^q}{q!}\,t^n$$

$$= \sum_{p,q=0}^{\infty} P_{p+q}(u,v)\,\frac{(w_1+zt)^p}{p!}\,(\alpha)_p h_q^{(\gamma,\beta)}(w_2+yt,xt^2). \qquad (2.18)$$

Remark 2.4. Choosing $w_1 = -zt$ and $w_2 = -yt$ in (2.1) and (2.2) , we deduce the following interesting corollaries:

Corollary 2.1.

$$\sum_{p,q,n=0}^{\infty} H_{p+q+n}(u,v)\, S_n^{p,q,M}(x,y,z)\,\frac{(-zt)^p}{p!}\,\frac{(-yt)^q}{q!}\,t^n$$

$$= \sum_{l=0}^{\infty} H_{Ml}\,(u,v)\,A_{Ml,Ml,l}\,\frac{(xt^{M})^{l}}{l!}\,. \tag{2.19}$$

Corollary 2.2.

$$\sum_{p,q,n=0}^{\infty} P_{p+q+n}\,(u,v)\,S_{n}^{p,q,M}(x,y,z)\,\frac{(-zt)^{p}}{p!}\,\frac{(-yt)^{q}}{q!}\,t^{n}$$

$$= \sum_{l=0}^{\infty} P_{Ml}\,(u,v)\,A_{Ml,Ml,l}\,\frac{(xt^{M})^{l}}{l!}. \tag{2.20}$$

Remark 2.5. Choosing $M = 1, 2$ in (2.19), we get the following result:

Corollary 2.3.

$$\sum_{p,q,n=0}^{\infty} H_{p+q+n}\,(u,v)\,S_{n}^{p,q,1}(x,y,z)\,\frac{(-zt)^{p}}{p!}\,\frac{(-yt)^{q}}{q!}\,t^{n}$$

$$= \sum_{l=0}^{\infty} H_{l}\,(u,v)\,A_{l,l,l}\,\frac{(xt)^{l}}{l!}, \tag{2.21}$$

and

$$\sum_{p,q,n=0}^{\infty} H_{p+q+n}\,(u,v)\,S_{n}^{p,q,2}(x,y,z)\,\frac{(-zt)^{p}}{p!}\,\frac{(-yt)^{q}}{q!}\,t^{n}$$

$$= \sum_{l=0}^{\infty} H_{2l}\,(u,v)\,A_{2l,2l,l}\,\frac{(xt^{2})^{l}}{l!}. \tag{2.22}$$

Remark 2.6. Choosing $M = 1, 2$ in (2.20), we get the following result:

Corollary 2.4.

$$\sum_{p,q,n=0}^{\infty} P_{p+q+n}\,(u,v)\; S_n^{p,q,1}(x,y,z)\;\frac{(-zt)^p}{p!}\;\frac{(-yt)^q}{q!}\;t^n$$

$$=\sum_{l=0}^{\infty} P_l\,(u,v)\;A_{l,l,l}\;\frac{(xt)^l}{l!}, \tag{2.23}$$

and

$$\sum_{p,q,n=0}^{\infty} P_{p+q+n}\,(u,v)\; S_n^{p,q,2}(x,y,z)\;\frac{(-zt)^p}{p!}\;\frac{(-yt)^q}{q!}\;t^n$$

$$=\sum_{l=0}^{\infty} P_{2l}\,(u,v)\;A_{2l,2l,l}\;\frac{(xt^2)^l}{l!}, \tag{2.24}$$

where $S_n^{p,q,M}(x,y,z)$ is the extended Srivastava polynomials (1.3).

3. Applications

I. In (2.21) and (2.23), choosing $A_{l,l,l}=(\alpha)_l$ and using (2.9), we get

$$\sum_{p,q,n=0}^{\infty} H_{p+q+n}\,(u,v)\;(\gamma)_p(\beta)_q\;g_n^{(\alpha,\beta+q,\gamma+p)}(x,y,z)\;\frac{(-zt)^p}{p!}\;\frac{(-yt)^q}{q!}\;t^n$$

$$=\sum_{l=0}^{\infty}(\alpha)_l\,H_l\,(u,v)\;\frac{(xt)^l}{l!}, \tag{3.1}$$

and

$$\sum_{p,q,n=0}^{\infty} P_{p+q+n}\,(u,v)\;(\gamma)_p(\beta)_q\;g_n^{(\alpha,\beta+q,\gamma+p)}(x,y,z)\;\frac{(-zt)^p}{p!}\;\frac{(-yt)^q}{q!}\;t^n$$

$$= \sum_{l=0}^{\infty} (\alpha)_l \, P_l \, (u, v) \, \frac{(xt)^l}{l!}. \tag{3.2}$$

Using relation (1.9) in the L. H. S. of result (3.1), we get:

$$\sum_{p,q,n=0}^{\infty} H_{p+q+n} \, (u, v) \, (\alpha)_p (\beta)_q \, g_n^{(\alpha,\beta+q,\gamma+p)}(x, y, z) \, \frac{(-zt)^p}{p!} \, \frac{(-yt)^q}{q!} \, t^n$$

$$= (1 - 2xyt)^{-c} \, F \, {}^{2:0;0}_{0;0;0} \left[\begin{matrix} \frac{c}{2}, \frac{c}{2} + \frac{1}{2} : -; -; \\ - \quad : -; -; \end{matrix} \; \frac{-4x^2t^2}{(1-2xyt)^2}, \frac{-4y^2t^2}{(1-2xyt)^2} \right], \tag{3.3}$$

and using relation (1.11) in the L. H. S. of results (3.2), we get:

$$\sum_{p,q,n=0}^{\infty} P_{p+q+n} \, (u, v) \, (\alpha)_p (\beta)_q \, g_n^{(\alpha,\beta+q,\gamma+p)}(x, y, z) \, \frac{(-zt)^p}{p!} \, \frac{(-yt)^q}{q!} \, t^n$$

$$= (1 - xut)^{-\alpha} \, {}_2F_1 \left[\frac{\alpha}{2}, \frac{\alpha + 1}{2}; 1; \frac{(xt)^2(u^2 - v - 1)}{(1 - xut)^2} \right]. \tag{3.4}$$

II. In (2.22) and (2.24), choosing $A_{2l,2l,l} = (\beta)_l$ and using (2.10), we get

$$\sum_{p,q,n=0}^{\infty} H_{p+q+n} \, (u, v) \, (\alpha)_p (\gamma)_q \, u_n^{(\alpha+p,\beta,\gamma+q)}(x, y, z) \, \frac{(-zt)^p}{p!} \, \frac{(-yt)^q}{q!} \, t^n$$

$$= \sum_{l=0}^{\infty} (\beta)_l \, H_{2l} \, (u, v) \, \frac{(xt^2)^l}{l!}, \tag{3.5}$$

and

$$\sum_{p,q,n=0}^{\infty} P_{p+q+n} \, (u, v) \, (\alpha)_p (\gamma)_q \, u_n^{(\alpha+p,\beta,\gamma+q)}(x, y, z) \, \frac{(-zt)^p}{p!} \, \frac{(-yt)^q}{q!} \, t^n$$

$$= \sum_{l=0}^{\infty} (\beta)_l P_{2l} \, (u, v) \, \frac{(xt^2)^l}{l!}. \tag{3.6}$$

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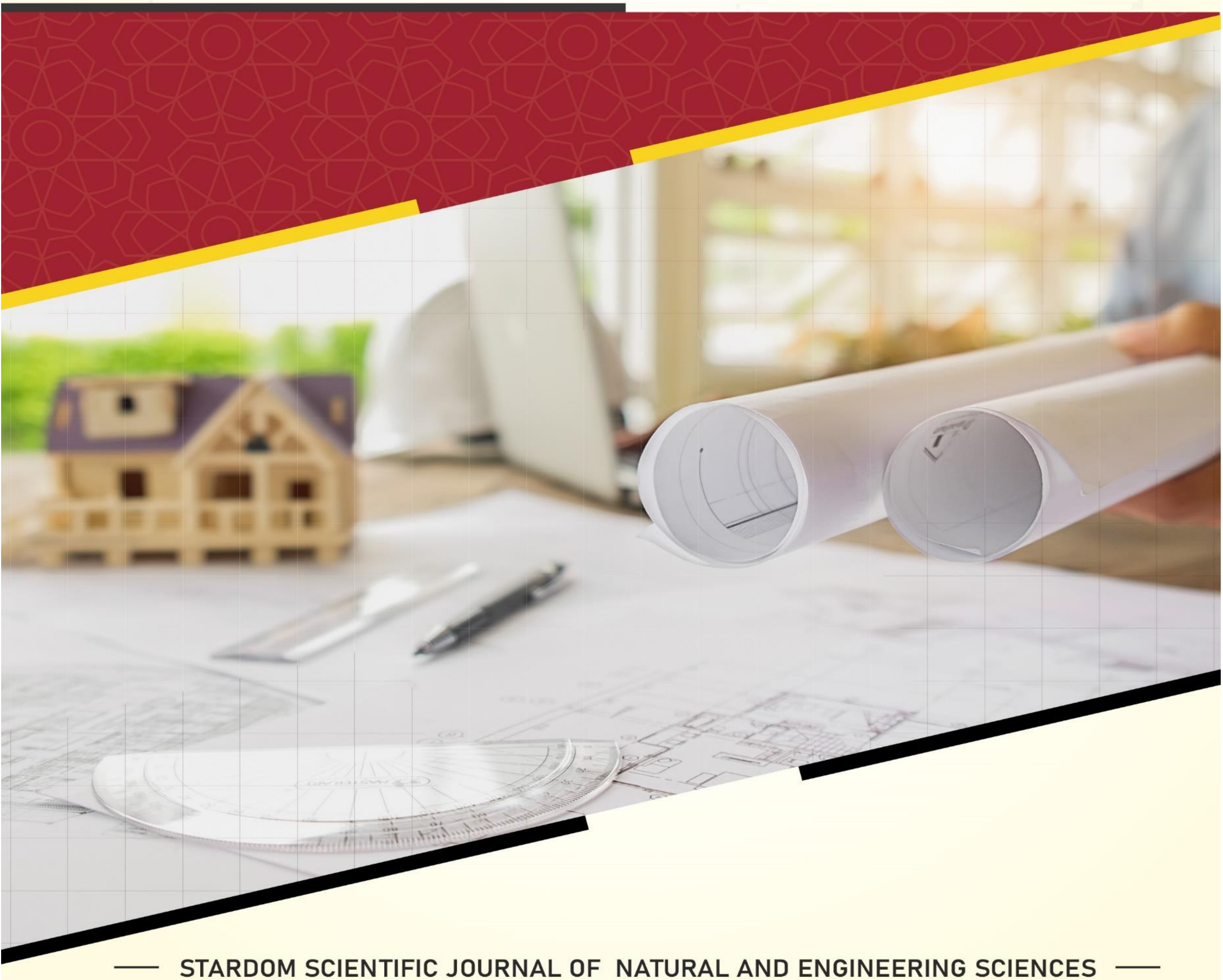

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